THE
AMERICAN
MATHEMATICAL MONTHLY

VOLUME XX JUNE, 1913 NUMBER 6

INCENTIVES TO MATHEMATICAL ACTIVITY.

By H. E. SLAUGHT, University of Chicago.

Sources. During the past ten years there has been a marked activity among teachers of mathematics in this country, looking toward improvement in methods and the reconsideration of subject matter, especially with reference to selection and emphasis. In this activity we are by no means alone. In fact, there has probably been still greater readjustment on the other side of the water. Doubtless the so-called Perry movement in England may be considered as one of the chief awakening influences underlying our own activity. But Professor Moore's presidential address[1] before the American Mathematical Society in December, 1902, "On the Foundations of Mathematics," may have been the real source of inspiration in this country which led to the special period of activity beginning at about that time.

The International Commission. As a world movement, all the activities in the various civilized countries were finally centered in the International Commission on the Teaching of Mathematics appointed by the Fourth International Congress of Mathematicians at Rome in 1908, on the motion of Professor D. E. Smith, of Columbia University. For four years this Commission occupied itself with collecting and publishing the fullest possible information as to the present status of the teaching of mathematics in each of the countries represented, not making any attempt thus far to propose or discuss reforms of any kind. The first report of this commission was made at the Fifth International Congress[2] held in Cambridge, England, in August 1912, where twenty-seven countries were represented and one hundred and fifty printed reports were presented, leaving about fifty still to be completed. The work of the commission will be continued

[1] *Bulletin of the American Mathematical Society*, Vol. IX, May, 1903, pp. 402–424. This article contains numerous references to the literature on this subject.

[2] For a general report of this Congress see the *Bulletin of the American Mathematical Society*, Vol. XIX, December, 1912, January, 1913. For a special report on the pedagogical aspects of the work of this Congress see THE AMERICAN MATHEMATICAL MONTHLY, Vol. XIX, October, 1912.

for another four years, giving opportunity for the study of numerous other topics not yet touched, and especially for the analysis and coordination of the information contained in the reports already published, for instance, a summary of the large features of these reports which could be printed in form for general distribution. The reports for the United States have been published by the Bureau of Education at Washington and may be had upon application to the Commissioner of Education.

Other Organizations. The work of the International Commission, however, must be considered as the outgrowth of activities which have their underlying causes far beneath the surface. For instance, in this country during the ten year period above referred to, there have been organized practically all of the now existing associations[1] of teachers of mathematics, including many city and state organizations and three large bodies,—in New England, the Middle States, and the Central West respectively. Two journals of wide circulation, supported by these associations, have also been founded during this same period; namely, *School Science and Mathematics* published at Chicago and *The Mathematics Teacher* published at Syracuse, N. Y.

All these organizations have been keenly active, especially the three large associations. For the most part, they have been engaged in the study of existing conditions and of means of improvement, through the reports of committees based upon long and serious investigations. Among the more important of such reports have been; a geometry syllabus by the New England Association, an algebra outline by the Middle States Association, another by the Missouri State Association, outlines for both algebra and geometry by a state conference meeting annually at the University of Illinois, a syllabus of algebra by the Central Association which was printed and reprinted, to meet the demands for it, until its circulation reached more than 20,000, and a geometry syllabus and an outline of a course in unified mathematics also by the Central Association, both of which were widely distributed and studied. Without doubt all of these documents have stimulated wide interest in pedagogical questions relating to secondary mathematics, and one, at least, has produced a profound impression upon the selection and presentation of subject matter as reflected in the newer text-books, namely, the algebra report of the Central Association.

The National Federation. Another organization in the interest of national coöperation among associations of teachers of mathematics and allied sciences was undertaken four years ago, namely, the Federation of Teachers of the Mathematical and Natural Sciences, which includes a membership of about 2,500 acting through the associations above enumerated. It was the purpose of this federation to deal with questions of national scope and it has been effective in at least two important matters, namely, (1) a bibliography of science terms and (2) a national geometry syllabus by a committee of fifteen, seven representing universities and eight representing secondary schools, acting under the joint auspices of the Federation and the National Education Association. This

[1] For a partial list of these associations see *School Science and Mathematics* for March, 1913.

geometry report of eighty pages was published in full in *School Science and Mathematics* in 1911, and again in final form in the *Mathematics Teacher* in 1912, and also in the proceedings of the National Education Association for 1911 and 1912. It was distributed through the Bureau of Education in Washington and has had a circulation, in this country and abroad, of 27,000 copies, and a request has just been granted to the Irish Journal of Education for its republication there in full.

The Underlying Causes. This decade of remarkable activity on the part of the teachers of mathematics in this country can be accounted for only on the hypothesis of a widespread and deep-seated feeling of unrest and dissatisfaction with the present methods and results, a conviction that in some way the mathematics teaching in the schools fails to connect vitally with the preparation for living in the twentieth century needed by the vast majority of the pupils in the schools. Possibly the following quotation from Professor Moore's address, above cited, may be the true analysis of the situation:

"He (Perry) asserts as essential that the boy should be *familiar* (by way of experiment, illustration, measurement, and by every possible means) with the ideas to which he applies this logic; and moreover that he should be thoroughly *interested* in the subject studied. . . .

"As a pure mathematician, I hold as the most important suggestion of the English movement the suggestion of Perry's just cited, that by emphasizing steadily the practical sides of mathematics, that is, arithmetic computations, mechanical drawing and graphical methods in general, in continuous relations with problems of physics and chemistry and engineering, it would be possible to give very young students a great body of the essential notions of trigonometry, analytic geometry, and the calculus. This is accomplished on the one hand by the increase of attention and comprehension obtained by connecting the abstract mathematics with the subjects which are naturally of interest to the boy, so that, for instance, all the results obtained by theoretic process are capable of check by laboratory process, and on the other hand by a diminution of emphasis on the systematic and formal sides of the instruction in mathematics. Undoubtedly many mathematicians will feel that this decrease of emphasis will result in much if not irreparable injury to the interests of mathematics. But I am inclined to think that the mathematician with the catholic attitude of an adherent of science in general (and at any rate with respect to the problems of the pedagogy of elementary mathematics in particular there is no other rational attitude) will see that the boy will be learning to make practical use in his scientific investigations, to be sure in a naïve and elementary way, of the finest mathematical tools which the centuries have forged, that under skillful guidance he will learn to be interested not merely in the achievements of the tools but in the theory of the tools themselves, and that thus he will ultimately have a feeling toward his mathematics extremely different from that which is now met with only too frequently—a feeling that mathematics is indeed itself a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions."

Whether or not this is the true interpretation of the present conditions, it is certainly significant that Professor Klein advocates practically the same remedy and that this is one of the questions which the International Commission has set for itself in its further deliberations.

The Indirect Results. Whatever may be the true causes of this widespread agitation and discussion, and whatever may be the final conclusions reached, this much is certain, that teachers of mathematics, the world over, are doing a vast amount of reading, investigating, experimenting, and careful thinking about the problems that confront them, and in this very attitude of mind lies the hope and the assurance that ultimately sane conclusions will be reached. In this country, especially, where there is a minimum of centralized authority, the final

disposition of these important questions depends largely upon such a development of thoughtful attention and investigation on the part of the general body of teachers as seems to be now in progress.

The indirect result, therefore, of all these discussions has been to stir teachers out of their lethargy and to encourage independent study outside of their routine duties. This seems to be the most plausible explanation of the vast increase in attendance upon the summer sessions at all the institutions where these are held. The report upon this topic[1] by the present writer, as a part of the American report of the International Commission, showed that the development of summer schools (that is, of summer sessions of normal schools, colleges and universities, conducted on the same basis as the work in any other part of the year), has been a remarkable phenomenon, and this has gone forward by leaps and bounds ever since that report was made. Along with the rapid increase of summer attendance in general, and of students of mathematics in particular, has gone the enrichment of the curricula, especially in the number and variety of courses which may count toward higher degrees, and in the courses on the history and teaching of mathematics. All this cannot fail to produce a profound impression upon those who are availing themselves of these opportunities, to arouse new enthusiasm, to stimulate fresh activity, to develop intelligent reaction on all the questions which confront the teachers of mathematics in this country.

The College Situation. For the most part, the foregoing data and observations apply to the teaching of mathematics in the secondary schools. All of the associations mentioned are primarily for secondary teachers, although many of the most enthusiastic members are college teachers. It is true that the investigations of the International Commission include schools of all grades from the elementary schools to the universities, but so far almost no attention has been given to the teaching of mathematics in the colleges, although it will readily be admitted that the problem is not so very different there, especially in first two years. It is practically certain that every charge that has been made against the teaching of mathematics in the secondary schools may be made with equal force in regard to the Freshman and Sophomore courses in college. In fact, there may be wider room for desirable modification in such subjects as analytic geometry and the calculus than in elementary algebra and geometry, if we may judge by some of the more recent texts and by the present practice of not a few colleges and universities.

But how shall the rank and file of teachers of college mathematics learn of the modifications in progress and of the success or failure of experiments in other institutions. Clearly the only way is by intercommunication, which must be either personal and hence very limited, or through some general medium of publication. It is precisely in this capacity that the editors of the MONTHLY believe its usefulness should be developed, and they are ready to devote any reasonable amount of space to the discussion of questions of this character.

[1] Published in the AMERICAN MATHEMATICAL MONTHLY, Vol. XIX, July, 1912. Afterwards also printed with the other reports in the Government bulletins.

[2] For one important exception see an item in the Notes and News of this issue.

However, it would not seem wise that such a journal should be devoted entirely or chiefly to pedagogical discussions as such. It may, and often does, happen that the stimulus derived from reading an article not directly pedagogical in its nature may produce a reaction strongly beneficial on the pedagogical side. Such, for instance, may be the effect of articles like those of Professors Hedrick, Huntington and Coolidge in the present issue, and such without doubt has been the effect of Professor Cajori's "History of Logarithms" which has been running since January. Even those articles which appear too abstruse for a given reader may provide just the stimulus which he particularly needs, not only to insure his continued mental activity but to actually furnish a form of mental stimulus which may be essential to his successful work as a teacher. This is the reason for publishing in the MONTHLY such articles as those of Professor Lehmer in the May issue, of Dr. Miles in the April issue, and of Professor Dickson in the March issue. (See the reference to this question under Notes and News.)

Conclusion. While it is true that agitation does not necessarily mean progress, it is also true that there is seldom any progress without agitation. We confidently believe that the unprecedented activity among teachers of mathematics during the past decade has resulted, and will further result, in substantial progress. It should be a keen incentive to every teacher that he or she may have some active part in our nation-wide determination to reconsider the foundations of our teaching and to improve our methods wherever possible; and that in this effort we are allying ourselves with a world-wide movement toward the same end.

HISTORY OF THE EXPONENTIAL AND LOGARITHMIC CONCEPTS.

BY FLORIAN CAJORI, Colorado College.

V. GENERALIZATIONS AND REFINEMENTS EFFECTED DURING THE NINETEENTH CENTURY.

THE GENERAL POWER AND LOGARITHM.

We have seen that the general theory of a^b, where both a and b are complex numbers, was outlined by L. Euler in his *Recherches sur les racines imaginaries des équations* of 1749, and that this paper failed to command the attention of mathematicians. We shall see now that three quarters of a century later the theory of the general power was elaborated by mathematicians of Germany, England, France and the Netherlands. At the opening of the nineteenth century this subject appeared difficult to many, as may be inferred from a paper of A. Q. Buée, in which the author contends that $\sqrt{-1}$ signifies perpendicularity,[1] and is finally led to the conclusion that $(\sqrt{-1})^n = \pm n(\sqrt{-1})$. The difficulty of the subject appears also in a paper of Argand who in 1813 ventured the statement that the expression $(\sqrt{-1})^{\sqrt{-1}}$ "would offer the simplest example of a

[1] *Philosoph. Trans.* for the year 1806, London, 1806, p. 67.